Comparison of Pricing Approaches for Longevity Markets

Melvern Leung

Monash University

melvern.leung@monash.edu

September 15, 2016

Overview

Background

- Longevity Risk in Pensions and annuity providers.

Background

- Longevity Risk in Pensions and annuity providers.
- LAGIC in Australia vs Solvency II in U.K.

Background

- Longevity Risk in Pensions and annuity providers.
- LAGIC in Australia vs Solvency II in U.K.
- Longevity linked securities: Bonds, swaps, options.

Motivation

- Private sector wants to diversify their portfolio, annuity providers/pension funds wants to hedge their longevity risk. Win-Win.

Motivation

- Private sector wants to diversify their portfolio, annuity providers/pension funds wants to hedge their longevity risk. Win-Win.
- Create products which gives incentive for private sector to buy into.

Motivation

- Private sector wants to diversify their portfolio, annuity providers/pension funds wants to hedge their longevity risk. Win-Win.
- Create products which gives incentive for private sector to buy into.
- Finding a "fair price".

Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.

Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.
- Discuss each of the four approaches used to price an s-forward

Setup

- Mortality modeling and forecasting under the CBD-model with a state-space representation.
- Discuss each of the four approaches used to price an s-forward
- Compare the results obtained.

Cairns Blake and Dowd Model

Denote a 1 year death probability for a person currently aged x at time t by $q_{x,t}$, then this can be modeled via,

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_{1,t} + \kappa_{2,t}(x-\bar{x}). \tag{1}$$

We adapt this model to incorporate an error component in the measurement equation so that a state-space approach can be applied.

$$\ln\left(\frac{q_{x,t}}{1-q_{x,t}}\right) = \kappa_{1,t} + \kappa_{2,t}(x-\bar{x}) + \varepsilon_t \tag{2}$$

Cairns et al. (2006) suggests that $\kappa_{1,t}$ and $\kappa_{2,t}$ can be modeled by a 2-Dimension random walk with drift,

$$\begin{bmatrix}\kappa_{1,t}\\ \kappa_{2,t}\end{bmatrix} = \begin{bmatrix}\theta_1\\ \theta_2\end{bmatrix} + \begin{bmatrix}\kappa_{1,t-1}\\ \kappa_{2,t-1}\end{bmatrix} + \omega_t$$

Statespace framework

Our framework is as follows:

$$\boldsymbol{y_t} = \ln\left(\frac{{}_1q_{x,t}}{1-{}_1q_{x,t}}\right) = \begin{bmatrix} 1 & (x_1-\bar{x}) \\ 1 & (x_2-\bar{x}) \\ \vdots & \vdots \\ 1 & (x_n-\bar{x}) \end{bmatrix} \begin{bmatrix} \kappa_{1,t} & \kappa_{2,t} \end{bmatrix} + \begin{bmatrix} \boldsymbol{\varepsilon_t} \\ \vdots \end{bmatrix}, \quad (3)$$

$$\begin{bmatrix} \kappa_{1,t} \\ \kappa_{2,t} \end{bmatrix} = \begin{bmatrix} \theta_1 \\ \theta_2 \end{bmatrix} + \begin{bmatrix} \kappa_{1,t-1} \\ \kappa_{2,t-1} \end{bmatrix} + \omega_t. \quad (4)$$

Where $\boldsymbol{\varepsilon_t} \sim$ i.i.d $N(0, \sigma_\varepsilon^2)$ and $\omega_t \sim N(0, \Sigma_\omega)$.

Prior Choices

$$\pi(\sigma_\varepsilon^2) \sim I.G(a_\varepsilon, b_\varepsilon)$$
$$\pi(\theta) \sim N(\mu_\theta, \Sigma_\theta)$$
$$\pi(\Sigma_\omega|\Sigma_{11}, \Sigma_{22}) \sim I.W\left(\nu + 2 - 1, 2\nu \; diag\left(\frac{1}{\Sigma_{11}}, \frac{1}{\Sigma_{22}}\right)\right)$$
$$\pi(\Sigma_{kk}) \overset{i.i.d}{\sim} I.G\left(\frac{1}{2}, \frac{1}{A_k}\right) \quad \text{for} \quad k \in (1, 2)$$

Priors where chosen such that they had conjugate forms to their respective likelihoods. A hierarchical structure was chosen for Σ_ω was due to the biased caused from a regular Inverse-Wishart prior (Huang et al., 2013; Gelman et al., 2006). The hyper parameters were chosen such that the priors were non-informative.

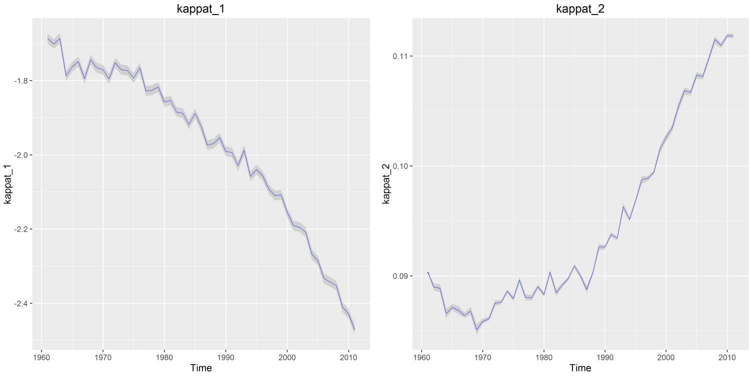
Summary Statitics

N=10000 draws, 3000 burn-in period, Australian dataset 1961-2011.

Table 1: Summary Statistics for parameters

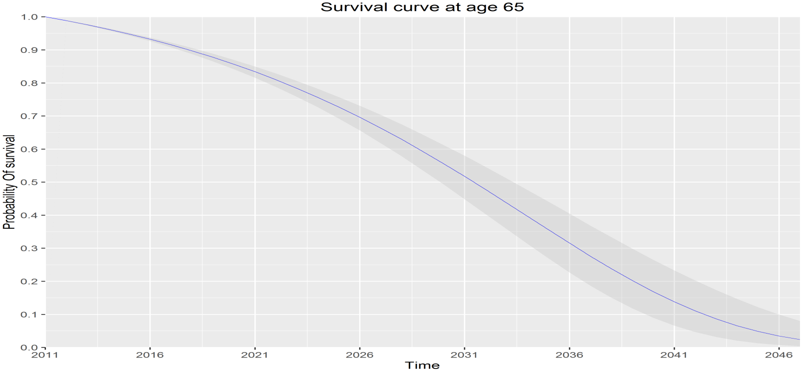
Parameter	Posterior Mean	95% HPD
θ_1	0.004793253	$(-0.02485228,\ -0.005813835)$
θ_2	0.000422871	$(0.000072951168,\ 0.0007697948)$
σ_ε	0.00263008	$(0.002458405,\ 0.002815683)$
Σ_{11}	0.001156807	$(0.0007116917,\ 0.001823677)$
Σ_{12}	$2.425724e-05$	$(1.084455e-05,\ 4.249324e-05)$
Σ_{22}	$1.56558e-06$	$(9.0352e-07,\ 2.516597e-06)$

Fitted curves κ_t



35-step ahead cohort direction forecast curve

Let k denote the k^{th} step ahead forecast, and N be the draws after the burn in period and $n = 1, ..., N$. Then, $\boldsymbol{\kappa}^{\boldsymbol{n}}_{\boldsymbol{T+k}} \sim N(\boldsymbol{\kappa}^{\boldsymbol{n}}_{\boldsymbol{T+k-1}} + \boldsymbol{\theta}^{\boldsymbol{n}}, (\Sigma_\omega)^n)$, $\boldsymbol{y}^n_{T+k} \sim N(\kappa_{T+k,1} + (x - \bar{x})\kappa_{T+k,2}, (\sigma^2_\varepsilon)^n \mathbb{I})$.



We look at 4 different pricing approaches:

1) Risk-neutral method (Cairns et al., 2006)
2) The 2-factor Wang transform (Wang, 2002)
3) Canonical valuation/ Maximum entropy method (Li and Ng, 2011)
4) An economic approach/ Tatonnement economics (Zhou et al., 2015)

The first two of these methods require data to find the risk-premium λ. Hence, we will use the issued but not sold EIB-bond to calibrate.

EIB bond

Using the setup for the EIB-bond, we apply Australian mortality projections to males aged 65 with a longevity spread of $\delta = 0.002$ over a $T = 25$ year period.

1) The price issued by EIB/BNP was in 2004, we assume that the prices have not been inflated since that time for 2011.
2) The setup was for England and Welsh males aged 65, we assume that the longevity spread δ would be kept the same for a population in Australia.
3) For ease of calculations, we assume a constant interest rate of 3%.

$$\bar{\Pi}_t(x, T) = \sum_{i=1}^{T} P(t, i) e^{\delta i} \hat{S}(x, i)$$

Under these assumptions, we find that the bond price $\bar{\Pi}_0 \approx 13.46739$

s-forward

Definition

An *s*-forward contract is a swap where the fixed rate payer pays an amount $K \in (0,1)$ in exchange for the realised survival probability ${}_T p_x$. An S-forward contract for a population aged x, over a maturity period T, will thus have a pricing formula given by:

$$SF(x,t,T,K) = P(t,T) E_{\mathcal{Q}} \left[{}_T p_x - K | \mathcal{F}_t \right].$$

Since an S-Forward contract has \$0 inception cost, we have to find the value of $K(T)$ such that the there will be no upfront cost.
$K(T) = E_{\mathcal{Q}} \left[{}_T p_x | \mathcal{F}_t \right]$

Pricing an s-forward

Under the 4 different pricing methodologies, if we were to price an s-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method, $K(T) = E_{\mathcal{Q}}\left[{}_Tp_x | \mathcal{F}_t\right] = \tilde{S}(x, T)$

Pricing an *s*-forward

Under the 4 different pricing methodologies, if we were to price an *s*-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
   $K(T) = E_{\mathcal{Q}}\left[{}_{T}p_x|\mathcal{F}_t\right] = \tilde{S}(x, T)$
2) Under Wang Transform Method,
   $K(T) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right]$

Pricing an *s*-forward

Under the 4 different pricing methodologies, if we were to price an *s*-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method,
$K(T) = E_{\mathcal{Q}}\left[{}_T p_x|\mathcal{F}_t\right] = \tilde{S}(x, T)$
2) Under Wang Transform Method,
$K(T) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right]$
3) Under Canonical Valuation method, $K(T) = {}_T p_x^{market}$

Pricing an *s*-forward

Under the 4 different pricing methodologies, if we were to price an *s*-forward, then the chosen $K(T)$ will be as follows:

1) Under Risk-neutral pricing method, $K(T) = E_{\mathcal{Q}}\left[{}_T p_x | \mathcal{F}_t\right] = \tilde{S}(x, T)$
2) Under Wang Transform Method, $K(T) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right]$
3) Under Canonical Valuation method, $K(T) = {}_T p_x^{market}$
4) Under the Tatonnement Approach, the value $K(T)$ is determined by the market under no risk-neutral assumptions. Hence, the value $K(T)$ will be found via an algorithm.

Risk-neutral pricing method

Our 2-D random walk with drift process:

$$\boldsymbol{\kappa_t} = \boldsymbol{\theta} + \boldsymbol{\kappa_{t-1}} + (\Sigma_\omega)^{\frac{1}{2}} Z$$

Where, $(\Sigma_\omega)^{\frac{1}{2}}(\Sigma_\omega)^{\frac{1}{2}} = \Sigma_\omega$ and $Z \sim N(0, \mathbb{I})$ is under real-world probability measure $\mathbb{P}$. Cairns et al. (2006) suggested that similar to the continuous time case, we can convert to the risk-neutral density (equivalent martingale measure) by,

$$\tilde{Z} = \lambda + Z,$$

Where, λ is the market price of longevity risk. Then,

$$\boldsymbol{\kappa_t} = \boldsymbol{\kappa_{t-1}} + (\boldsymbol{\theta} - (\Sigma_\omega)^{\frac{1}{2}}\lambda) + (\Sigma_\omega)^{\frac{1}{2}}\tilde{Z}$$

Risk-neutral pricing method

Under risk-neutral assumption the EIB-bond price is given by:

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) E_{\mathcal{Q}(\lambda)} \left[e^{-\int_t^i \mu_x(u)du} | \mathcal{F}_t \right]. \qquad (5)$$

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, i, \lambda)$$

Market Price of Risk	Value	$\bar{\Pi}_0(65, 25)$
(λ_1, λ_2)	$(0.27307,\ 0.27307)$	13.46739
(λ_1, λ_2)	$(0.24505,\ 0)$	13.46739

2-factor Wang Transform

Wang (2002) gave a universal pricing method, such that, assume we have a liability X over a time period $[0, T]$ with $F_X(x) = P(X < x)$, then with a market price of risk λ, the risk-adjusted (distorted) function of $F(X)$ can be found by,

$$F^*(x) = Q\left(\Phi^{-1}(F(x)) + \lambda\right)$$

Where, $F^*(x)$ is the risk-adjusted function for $F(x)$. Since our aim is to find longevity risk, we have,

$$\tilde{S}(x,t) = E\left[Q\left(\Phi^{-1}(S(x,t)) + \lambda\right)\right] \text{ for } t \in [0, T]$$

Where $Q \sim Student - t(\nu)$.

2-factor Wang Transform

To find λ using the EIB-bond

$$\tilde{\Pi}_t(x, T, \lambda) = \sum_{i=1}^{T} P(t, i) Q\left(\Phi^{-1}(S(x, T)) + \lambda\right). \qquad (6)$$

Matching the price at initial time $t = 0$,

$$\sum_{i=1}^{T} P(0, i) e^{\delta i} S(65, i) = \sum_{i=1}^{T} P(0, i) \tilde{S}(65, T)$$

Market Price of Risk	Value	$\bar{\Pi}_0(65, 25)$
λ	0.3478043	13.46739

Canonical Valuation

The maximum entropy principle was first proposed by Stutzer (1996) and used by Kogure and Kurachi (2010); Foster and Whiteman (2006) to find market survival probability denoted by ${}_T p_x^{\text{market}}$. In our case we by using the EIB-Bond in combination with the maximum entropy principle to find ${}_T p_x^{\text{market}}$.

Canonical Valuation methodology

1. $\boldsymbol{p_x^j} = ({}_1p_x^j,\ {}_2p_x^j, \ldots,\ {}_Tp_x^j), \quad$ for $j = 1, \ldots, N$. and let $\boldsymbol{\pi}$ denote the empirical distribution for $\boldsymbol{p_x}$
2. $\bar{\Pi}$ denote the market price of the EIB-bond $\bar{\Pi}(65, 25)$.
3. Let $\boldsymbol{\pi}^*$ be the risk-neutral distribution for $\boldsymbol{\pi}$, then $\sum_{j=1}^N \Pi^j \pi_j^* = \bar{\Pi}$.
4. Then the maximum entropy principle stipulates that, $\boldsymbol{\pi}^*$ should minimize the Kullback-Leiber Information divergence, $\sum_{j=1}^N \pi_j^* \ln\left(\frac{\pi_j^*}{\pi_j}\right)$, subject to the constraint $\pi_j^* > 0$ and $\sum_{j=1}^N \pi_j^* = 1$.

Canonical Valuation methodology

1. Kapur and Kesavan (1992) derived the solution to the minimization of $\sum_{j=1}^{N} \pi_j^* \ln\left(\frac{\pi_j^*}{\pi_j}\right)$ subject to $\bar{\Pi}$, is given by
$$\hat{\pi}_j^* = \frac{\pi_j \exp\left(\gamma \bar{\Pi}^j\right)}{\sum_{j=1}^{N} \pi_j \exp\left(\gamma \bar{\Pi}^j\right)}$$
2. Find γ via, $\bar{\Pi} = \frac{\sum_{j=1}^{N} \bar{\Pi}^j \exp\left(\gamma \bar{\Pi}^j\right)}{\sum_{j=1}^{N} \exp\left(\gamma \bar{\Pi}^j\right)}$.
3. $\sum_{j=1}^{N} {}_t p_x^j \pi_j^* = {}_t p_x^{\text{market}}$

In this scenario, we don't assume there is a risk premium λ, but there is a γ parameter which "corrects" the real world probability ${}_t p_x$to adjust for the market accepted ${}_T p_x^{market}$.

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an *s*-forward, θ_B is the demand.

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an *s*-forward, θ_B is the demand.
   - ω_t^A is the wealth of A, ω_t^B is the wealth of B at time t.

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an *s*-forward, θ_B is the demand.
   - ω_t^A is the wealth of A, ω_t^B is the wealth of B at time t.
   - Let the function $g(S(x,t))$ denote the gains of the *s*-forward at time t.

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an s-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an s-forward, θ_B is the demand.
   - ω_t^A is the wealth of A, ω_t^B is the wealth of B at time t.
   - Let the function $g(S(x,t))$ denote the gains of the s-forward at time t.
   - Let $f(S(x,t))$ represent the payout for the survival probability for the hedger at time t.

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an *s*-forward, θ_B is the demand.
   - ω_t^A is the wealth of A, ω_t^B is the wealth of B at time t.
   - Let the function $g(S(x,t))$ denote the gains of the *s*-forward at time t.
   - Let $f(S(x,t))$ represent the payout for the survival probability for the hedger at time t.
3. $\theta_A = \sup_{\theta_A} E\left[U\{\omega_{t-1}^A e^r - \theta^A g(S(x,t)) - f(S(x,t))\}\right]$

Tatonnement Approach

This was first suggested by (Zhou et al., 2015). In summary, we are trying to find an accepted strike price K, that is accepted by the market.

1. Assume we have a buyer (investor) of an *s*-forward and a seller (hedger).
2. Denote the following parameters:
   - θ_A is the supply of an *s*-forward, θ_B is the demand.
   - ω_t^A is the wealth of A, ω_t^B is the wealth of B at time t.
   - Let the function $g(S(x,t))$ denote the gains of the *s*-forward at time t.
   - Let $f(S(x,t))$ represent the payout for the survival probability for the hedger at time t.
3. $\theta_A = \sup_{\theta_A} E\left[U\{\omega_{t-1}^A e^r - \theta^A g(S(x,t)) - f(S(x,t))\}\right]$
4. $\theta_B = \sup_{\theta_A} E\left[U\{\omega_{t-1}^B e^r + \theta^B g(S(x,t))\}\right]$

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).
for each time period $t \in [1, T]$

1. Guess an initial K.

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).
for each time period $t \in [1, T]$

1. Guess an initial K.
2. Determine θ_A and θ_B.

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).
for each time period $t \in [1, T]$

1. Guess an initial K.
2. Determine θ_A and θ_B.
3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).

for each time period $t \in [1, T]$

1. Guess an initial K.
2. Determine θ_A and θ_B.
3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
4. else, update K by:

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).

for each time period $t \in [1, T]$

1. Guess an initial K.
2. Determine θ_A and θ_B.
3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
4. else, update K by:
   1) $K^{i+1} = K^i + h^i$

Tatonnement Approach

Since g is an s-forward, $g = (S - K)$, and choosing an Exponential Utility function, we use the algorithm suggested by (Zhou et al., 2015).
for each time period $t \in [1, T]$

1. Guess an initial K.
2. Determine θ_A and θ_B.
3. if $\theta_A = \theta_B$ then stop, and set $K(t) = K$
4. else, update K by:
   1) $K^{i+1} = K^i + h^i$
   2) $h^i = \gamma|K^i|(\theta_B - \theta_A)$

Results

For each of the methods, 1000 chains from the MCMC was used after burn-in. A monte-carlo average was taken when an expectation was involved. Using a a portfolio of aged 65, with a hedging period of $T = 5, 10, 15, 20, 25$ of the s-forward. The prices are shown below:

Period	real-world	Risk-Neutral	Wang-T	Canonical	Tat
$K(5)$	0.93233	0.93577	0.96720	0.93219	0.93225
$K(10)$	0.83398	0.84430	0.90622	0.83378	0.83398
$K(15)$	0.69621	0.72295	0.80547	0.69648	0.69621
$K(20)$	0.51740	0.56046	0.65226	0.52053	0.51740
$K(25)$	0.31560	0.37075	0.44740	0.32598	0.31790

Conclusion

- Bayesian methods allows us to have prediction uncertainty in a systematic way via the prior distribution.
- The different choices of methods, produced "similar" results, except for the tatonnement approach. Under economic conditions, it shows that there really isn't a need for a "premium" if both investor and hedger acts "rationally".
- Under the Wang transformed density, the premium is much higher than other methods. This is because the effect of the distortion operator causes a greater change in mortality directly compared with the risk-neutral method.

The End

Markov-Chain Monte-Carlo (MCMC)

Let $\psi = (\sigma_{\varepsilon}^2, \Sigma_{\omega}, \boldsymbol{\theta})$. An MCMC method will be used to explore the posterior distribution and parameter states.

- Obtain initial draws denoted by ψ^0.
- Conditional on ψ^0, find the distribution of latent states via the Kalman Filter.
- latent variable $\kappa_{1:T}$ drawn recursively from Backward Sampling (Carter and Kohn, 1994).
- Conditional on drawn latent variable, draw model parameters from their respective conditional posterior density.

Cairns, A., Blake, D., and Dowd, K. (2006). A two-factor model for stochastic mortality with parameter uncertainty: Theory and calibration. *Journal of Risk and Insurance*, 73(4):687–718.

Carter, C. K. and Kohn, R. (1994). On gibbs sampling for state space models. *Biometrika*, 81(3):541–553.

Foster, F. D. and Whiteman, C. H. (2006). Bayesian prediction, entropy, and option pricingx. *Australian Journal of Management*, 31(2):181–205.

Fung, M. C., Ignatieva, K., and Sherris, M. (2015). Managing systematic mortality risk in life annuities: An application of longevity derivatives. *UNSW Business School Research Paper*, (20015ACTL04).

Gelman, A. et al. (2006). Prior distributions for variance parameters in hierarchical models (comment on article by browne and draper). *Bayesian analysis*, 1(3):515–534.

Huang, A., Wand, M. P., et al. (2013). Simple marginally noninformative prior distributions for covariance matrices. *Bayesian Analysis*, 8(2):439–452.

Kapur, J. N. and Kesavan, H. K. (1992). *Entropy optimization principles and their applications*. Springer.

Kogure, A. and Kurachi, Y. (2010). A bayesian approach to pricing longevity risk based on risk-neutral predictive distributions. *Insurance: Mathematics and Economics*, 46(1):162–172.

Li, S.-H. and Ng, C.-Y. (2011). Canonical valuation of mortality-linked securities. *Journal of Risk and Insurance*, 78(4):853–884.

Stutzer, M. (1996). A simple nonparametric approach to derivative security valuation. *The Journal of Finance*, 51(5):1633–1652.

Wang, S. S. (2002). A universal framework for pricing financial and insurance risks. *Astin Bulletin*, 32(02):213–234.

Zhou, R., Li, J. S.-H., and Tan, K. S. (2015). Economic pricing of mortality-linked securities: A tatonnement approach. *Journal of Risk and Insurance*, 82(1):65–96.